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MATHEMATICAL EDUCATION AND SOCIETY

(an outlook from Russia and into Russia)

Igor F. Sharygin

Abstract. The author gives surprising examples of mathematical ignorance of modern society. One can say that people nowadays not only are ignorant of or do not understand elementary mathematical concepts but that they are afraid of mathematics. Mathematicians are not last who should be blamed for this. Among other problems that impede the development of mathematics education, the author singles out the following three: Americanisation, dollarisation and computerization.

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An ellipse is a circle inscribed in a square 7×8.

Folklore

When a well-known writer was asked a question why he watched TV, he answered: "Where else can I see so many idiots at once?" For a person with mathematical education it is sometimes unbearable to watch TV or read newspapers. It is impossible to observe calmly general mathematical illiteracy coming down from TV and newspapers.

Here are two illustrative examples. Once I caught sight of a popular TV program and heard a dialogue between a TV-man and a well-known singer. "Do you know what a circle and the diameter of a circle are?" – asked the TV-man. "Yes, of course." – answered the singer. "How many times approximately is the length of the circumference of a circle greater than the length of the diameter?" "About fifteen times." "It is wrong, of course! There are 3 answers: 1) about three times; 2) about five times; 3) about ten times. Which answer is correct?" "The last one." – was his immediate reply. "Do you agree with that answer?" – the TV-man asked a young girl who was a student of the faculty of economics. "Oh, yes!"

The second example. My son was watching another popular TV program. It was a game "The Keys to the Fort Bayard" (a French television broadcasting). One episode from the TV show attracted my attention. A young lady was competing with the master of the game. There were 21 sticks on the table. The participants had to take one, two or three sticks in sequence. According to the rule of the game it would be lost if one of the participants took the last stick. The master was the first to start. The young lady was allowed to repeat his moves. At the end of the game there were 5 sticks left. The master took 3 of them and the girl the remaining 2 sticks. So, she lost the game. The young lady and other members of her team were upset.

Both of these examples demonstrate not only the lack of general mathematical literacy. It is a pathology. I will not give any additional examples and pass on to the main body of the text.

There has been a tremendous growth of mathematics and mathematical education in the 20th century. The history of mathematical education itself with reference to Russia actually began in the 20th century or, more precisely, at the end of the 19th century. The achievements of the Soviet-Russian mathematical science and mathematical education are universally known and generally recognized. But today we are painfully aware of the sad state of mathematical education in Russia. There are objective reasons for the deterioration of our mathematical culture and they are caused by the general economic crisis in Russia. But the mathematicians are to be blamed for this degradation as well. It is very important to realize and analyse the strategic and tactical errors made by the mathematicians, scientists and methodologists that have influenced the decline in mathematical culture and the level of mathematical education in our country.

The first mistake: **violation of the principle of a historical method**.

In mathematical science the end of the 19th century and the first half of the 20th are characterized by laying the foundations of mathematics. All of a sudden the mathematicians began to discover numerous logical gaps in the mathematical foundation and tried to bridge these gaps thoroughly. In doing so, they decided to make a mathematical course in school logically faultless (from a formal mathematical point of view) and first of all, a geometry course which was a kind of medieval survival against the background of technological advances and new innovative educational programs for other subjects. Thus, almost the latest advances in mathematical science have been integrated into the school curriculum.

Now we realize that the educational process is governed by basic biological laws and it is impossible to accelerate it much like a process of bearing a child. During the course of studies more attention should be given to the history of mathematics, since throughout the history mathematics has not only reflected developments in civilization but also made a major contribution to those developments. Contrary to other sciences, first of all, natural sciences there were no (or almost no) erroneous theories in mathematics. Certainly, it is unreasonable now to study geocentric theory of the universe (though I am not sure about it, an embryo in the process of development passes through a branchiate stage, you see.)

The history of development of mathematics gives us optimal mathematical curriculum. Procedure of optimisation is done by history itself.

But before coming to the next point it should be mentioned that the mathematicians have not solved the problem of constructing a perfectly logical foundation of mathematical science yet. Even among themselves the mathematicians cannot come to an agreement about what is valid and proved and what is not, but nevertheless some of them are trying to impose on school rather complex and controversial logical ideas. Presently there is much speculation that instead of meticulous analysis of the principals of foundation there must be a more realistic approach because mathematics in its basis is closer to natural sciences, such as physics.

The second mistake: **a keen interest in the so-called "left hemisphere mathematics" and "left hemisphere methods of instruction"**.

It is common knowledge that the majority of mathematicians have a special type of intelligence, namely, the logical-mathematical one. According to recent researches in the field of physiology and psychology the left and the right cerebral hemispheres perform different functions. The left side of the brain is responsible for logical thinking, whereas the right side controls visual perception and sensuality. Hence, the right-side hemisphere is primary and the development of the left-side hemisphere is conditioned by a prolonged period of evolution. In our sleep the left hemisphere has a rest, but the right hemisphere is always functional. Dreams are products of the right side of the brain. The left side of the brain is more powerful than the right side. Presumably, a well-developed left hemisphere is a distinguishing characteristic of a professional mathematician. But I am of the opinion that the potential of a creative personality (an artist, a poet, a scientist, a mathematician as well, or an inventor) is determined by the development of the right hemisphere of the brain. Nevertheless, there are quite a lot of mathematicians and scientists with hypertrophied left hemisphere. In everyday life such individuals tend to have algorithmic thinking, inferiority complex and even emotional problems. And it is precisely these individuals have come to the forefront in the field of mathematical education in our country (and abroad?) Their activities do mathematical education and teaching techniques a disservice, since their recommendation have produced unfavourable effect upon the curriculum and evaluation standards for school mathematics. Luckily, the human race, on the other hand, consists largely of a different type people.

Although most people are right-handed, some are left-handed. Not long ago, parents and teachers used to try and change left-handed children into right-handed. Today scientists know it is best for left-handed children to stay the way they are. Forcing them to use their right hand confuses the two sides of the brain. It is worse when the specialists with hypertrophied left hemisphere of the brain are trying to launch innovative mathematical programs whereby mathematics is taught as a collection of unjustified definitions, logically unsound arguments (what a paradox!), trivial propositions, in which a mere combination of empty words is taken for inference. As a result many children are turning away from mathematics altogether. We must be aware of the fact that "such mathematics" will certainly not be a favourite study in school. Moreover, pupils can develop even a distaste for and fear of mathematics. Such an attitude towards teaching mathematics is irrational and it may well influence the mathematical development of our youth and eventually bring about a lack of leadership in science.

Recently, I have reviewed a textbook on Geometry which took the first place in the Russian competition of textbook sponsored by the World Bank. (I think that the World Bank sponsors acceleration of degradation of our education.) Among various problems there was the following: "Absolutely clear that a line segment has only one midpoint. But how to prove it?" (By the way, there are textbooks in which this problem is proved like a theorem.) I could not prove this statement

but I was helped by the author. Another example from the same textbook was concerned with the construction of the bisectors. There was an explanation that "this problem has one, and only one, solution, otherwise the two halves of the same angle would not be equal." This explanation bears a resemblance to one little girls' reasoning from a record by Lewis Carroll: "It is so wonderful that I dislike asparagus, because if I liked it I would have to eat it. And I detest it."

Naturally, these "theorems" and "proofs" are rejected by children on the basis of common sense. But their attitude gives the authors of new textbooks (usually the same authors) a wrong idea that a great number of children do not understand mathematical arguments and it would be better to omit them from the textbooks. As a consequence in newly written textbook a lot of arguments are excluded from consideration. But a mathematical textbook without arguments is like a swimming pool without water.

But unfortunately, many attempts of competent specialists in school mathematics to revise textbooks have not been successful so far since their initiatives have been ignored by educational administration.

V. I. Arnold in his report ("Antiscientific Revolution and Mathematics" at the International Congress of Papal Academy of Science in Vatican, October 26, 1998) said the following: "Those who did not master the art of rigorous mathematical reasoning in school fail to distinguish between true and false reasoning. The irresponsible politicians can easily manipulate such people. The most unexpected outcome may be a state of hypnosis on a mass scale and social disturbances".

This is exactly what is happening in Russia now (and not only in Russia).

The third mistake: **misinterpretation of the relationship between mathematical science and mathematical education**.

School mathematical education has often attracted considerable attention of distinguished mathematicians. Undeniably, it is very good. But on the other hand, professionals in pure mathematics usually consider school mathematics as an integral part of mathematical science its initial stage. And it is a grave fallacy. Having indisputable authority and enormous knowledge in their own field these prominent scientists have frequently invaded a domain of public education in which they are not always competent (another false belief that it is quite enough to be a good mathematician to solve the problems of mathematical education) and using their authority have often overwhelmed the inert masses of school teachers. Note, that in education the reforms from above (the board of education) are rather dangerous. In my opinion, the reform initiated by a remarkable scholar A. N. Kolmogorov has adversely affected our Soviet-Russian mathematical education and caused serious damage to it.

Something similar happened in France, where an outstanding mathematician Dieudonné became one of the main ideologists of school reforms. It was Dieudonné who brought forward an idea (speaking about school Geometry): "Euclid must go away". In his view Euclidean Linear algebra should be substituted for Geometry.

Notice, I partly agree with his proposal but for a different reason. Euclid's "Elements" was the first successful attempt for his time as well as for ours to

create a geometrical theory based on axioms. Even nowadays, the logical level of "Elements" greatly exceeds the potentialities of an average pupil and not only average.

The fourth mistake: **the lack of adequate attention to the subject-matter of mathematics in primary and junior school**.

Participation of prominent mathematicians in the school reforms were restricted to high school only. Primary and junior schools were outside the field of vision of professional mathematicians. The direct consequence of this neglect is a great number of new textbooks written by almost illiterate authors (from a mathematical point a view). These authors are convinced that their knowledge of the multiplication table and a few logical rules suffices to write textbooks for primary school. Besides, they are trying enthusiastically to introduce formal ideas of the "left hemisphere" mathematics into primary and junior schools. They torment poor children by making them prove commutativity of addition of integers, demanding explanation why the sum of two even numbers is even, etc. It is a caricature of mathematical logic and of an axiomatic method and not a benign one.

It is not improbable that most if not all people are born with the better developed right side of the brain and in early childhood the left side of the brain always develops more slowly than the right one. Therefore, the study of the "left hemisphere" mathematics according to the corresponding methods in primary school may have disastrous consequences.

Linking the fourth mistake with the first I will formulate my own point of view: training and a mental development of a child in a primary school must be determined by triad (three closely related things): number, form, word. Geometric activity is an initial stage of mental activity, both historically (for all mankind) and genetically (for an individual). And it is very important to arrange teaching process in an orderly sequence, especially today when many of the traditional kinds of children activities developing geometrical imagination have been lost. Formally, mathematical education begins with arithmetic. The arithmetic textual tasks are significant not only for arithmetic-logical development of a child, but they are closely connected with geometry as well. And it seems plausible that disappearance of the traditional textual arithmetic tasks from primary school curriculum may account for the deterioration of geometric culture in school.

The fifth mistake: **inability and unwillingness to popularise mathematical knowledge to improve the "image" of mathematics in the public eye**.

Mathematicians shutting themselves off from society and hiding behind a thin but rather hard frontal bone of the forehead practically do not advocate mathematical knowledge, its essence, and its importance for the advancement of all public institutions. It is not surprising then, that the laymen have a distorted idea about mathematics. Have you ever seen a prominent mathematician giving an interview or just making remarks before TV camera? In what television broadcasting can you hear a good word in favour of mathematics? Or, in what newspaper can you read an interesting and popular article about the role of mathematics?

Mathematicians do not use in their own interests even a physiological necessity

which is common to each Homo sapiens (still being alive human species) to provide their brain with "food for thought". This basic need is often realized by their participating in idiotic TV contests, where the participants exercise their mind guessing words or riddles, taking part in the "so-called" intellectual games. (In such contests intellect is evaluated in money. An ordinary man is suggested a simple idea "the more money he has, the cleverer he is".)

The sixth mistake: **lack of attention to the methods of teaching (methodology)**.

Pedagogical science as a whole and its mathematical branches concerning methods of teaching mathematics and of instruction are developing, to say it more correctly, functioning without any assistance of mathematicians who do not even consider pedagogy to be a branch of science at all. Perhaps, they are right thinking of pedagogics like this, because there are no necessary corresponding indications that it is a science. For instance, there always exist scientific discoveries in any branch of science. I do not know a single discovery made by a prominent educator in the field of pedagogy. Even in cookery it is possible to invent a new recipe of mayonnaise. There is no even "mayonnaise" in pedagogy.

Just the same, professional mathematicians should not keep away from the problems concerning methods of teaching. Their great intellectual potential can protect school teachers against mathematically uneducated specialists in pedagogy along with the officials from educational administration.

One can easily observe some contradictions among points 3, 4 and 6. On one hand, the author points to some unfavourable consequences of an intrusion of professional mathematicians into school, and on the other hand, he calls on them to assist school more efficiently. I agree that there is some contradiction.

It is the concern of the professional in mathematics to help a school teacher find a path between the mire of ignorance and the thicket of mathematical subtleties, prevent a school child from sinking to the bottom of mathematical illiteracy and keep him from getting lost in the above-mentioned thicket.

The seventh point (a reproach to the educational administration rather than to mathematicians): **enthusiasm for "innovative" programs, preference to foreign experience, commercialisation of education**.

One of the traditions existing in Russia is disrespect for its own customs and traditions and constant admiration of foreign experience at all times. Nowadays there is a passion for all kinds of innovations and accordingly innovators in education as it was during the Soviet period (and, probably, earlier as well).

The Russian mathematical education is presently exposed to many dangers, but the most serious of them according to V. I. Arnold (and myself) is "Americanisation" of the Russian mathematical education. It has been discussed many times that the standard of the American education is rather low. I will not give generally known facts. I will restrict my consideration to a single example from the American source. In "The American Mathematical Monthly" (November, 1998) there is an article written by Alfred Manaster concerning a survey (in the context of TIMSS) of the quality of mathematical education in the 8th grade of high school in three

countries: Japan, Germany and USA. Here are some conclusions from this article. Mathematical reasoning was employed in the lessons of mathematics as follows: in Japan—in 53% of all the lessons attended, in Germany—in 20% of the lessons and in USA—in 0%. For the most part, mathematical reasoning was used at the lessons of geometry which is scarcely presented in school curriculum in USA. The curriculum and evaluation standards for school mathematics in USA lag behind the corresponding programs in Japan and Germany approximately by two years. The schoolchildren of the 8th grade in USA study mainly the beginnings of algebra ("Before Algebra") while the schoolchildren of the same age in Japan and Germany have it studied earlier.

Paradox is that. American mathematical education itself is one of the worst in the world, but the system of American mathematical education as a whole is probably the best, since the whole world is integrated into it. Despite the repeated attempts of some officials from the Board of Education to promote the American style of teaching, mathematical community in Russia is still resistant to it. The potential danger of "Americanisation" lies not only in the fact that American mathematical education is one of the worst in the world. (However, it is not an obstacle to advancement of American Mathematics which is according to Napoleon's well-known remark closely connected with the welfare of society.) The problem is quite different. The main question regarding Russian mathematical education has always been: "Why?", whereas the chief question concerning American mathematical education is "How?" These questions are fully consistent with peculiarities of the national character. Instead of the American "know how" we have the Russian "know why". It is absolutely clear that in the line of this paradigm, we have two completely different systems of mathematical education, requiring different textbooks and accordingly different teaching instructions. It is also clear that the penetration of the American style into our mathematical education will bring (and has already brought) a serious internal conflict and as a result it may simply destroy our mathematical education. In time it will produce particularly grave consequences, particularly in the field of geometric education.

The American science is supported by "the world brains" contrary to the Russian science which is sustained by its own brains. The decline in mathematical education may destroy the Russian science once and for all.

People may fall ill as a result of a simple change of food. The sharp change of existing spiritual food and moral values of the whole nation may cause a serious mental disease in the whole society, a kind of split personality, and interfere with its development. It is unlikely that our society will be in good health in the nearest future. Thus, let us try to preserve the healthy organs which have not been affected yet, such as our mathematical education.

One of the most important problems facing education as a whole is preservation of genetic code, introduction of this code into personal genetic programs of new generations. It is precisely this fact that should be taken into consideration in evaluating all "innovations" which the officials from the Board of Education are fond of. Moderate nationalism and conservatism are prerequisite to the efficiency

of education system.

Preservation of national traditions in education, and not only in it, is beneficial to the all mankind as a whole, it sets up a difference in potential in human society and thus contributes to the development of Earth civilization. Because of this, making of global unified educational system is not a good idea, and some bring them to perfection, giving back to the originators in the form "know how". Suum cuique.

Speaking about "Americanisation" of our educational, in particular, mathematical education, I mean pragmatism which is characteristic of majority changes happening today and its specific manifestation in standards and tests. Many experts (even quite reasonable) agree with the necessity of changes in education exactly in direction of pragmatism using standards and tests. They disagree on how these ideas can be realized. (I believe that standards are one of the auriferous veins in exploiting of which the proponents of this idea are interested in as long as possible.)

But pragmatism is especially profitable when you live among idealists. Society consisting exclusively of pragmatists is lacking vitality as well as society including only idealists.

A peasant community has always made ample provision for three idealists: a clergyman, a teacher and a doctor. Not infrequently, the fourth one joined them—a village fool.

The "Hellenistic culture was destroyed by the Romans who were interested only in the ultimate result useful for military purposes, navigation and architecture." (from V. I. Arnold's report) But by doing so, the Romans lost a fertile ground for their pragmatism and entered a new phase which led to the collapse of the Roman empire.

"Americanisation" of society which is taking place in many countries now may result in the same destruction of science and culture of modern civilization. And in my own view it may be the beginning of the downfall of the American empire itself.

As you are well aware, the extreme value of a linear function is achieved at the boundary of variation. Nowadays, the criterion determining success of an individual and human societies is evaluated by a linear function—"a sum in dollars". So, a human society must approach the extreme point of the domain of its existence. This point can be passed by automatically with ease, and it is followed by destruction and decay. But an instinct of self-preservation of human societies is monotonously growing weak as the number of members of these societies is increased .

Today the classical Marxist formula "money – goods – money" has given way to a new formula "money – money – Money" and as quickly as possible. The chain: "money – education – science – industry" (and as a result, the same money) which is essential to a not malfunctioning and developing of any society does not obviously withstand competition with other much more profitable and rapid cycles.

"Dollarisation" of education puts in the forefront precisely the destruction of mathematics as the least "dollar capacity" subject. ("Dollar capacity" can be de-

termined as a ratio of the quantity of applied sum of money with respect to the quantity of efforts exerted.) And here the conventional techniques of teaching mathematics in our country are substantially exceeded by both, computer technologies (their numerator is much greater) and the technologies based on using tests (their denominator is much less).

The pragmatic Americans realized long ago that it is cheaper to buy a mathematically educated person than to train him.

Pragmatism and global computerization are a dangerous alliance. What for to learn to multiply or divide by hand if there is a computer at your disposal? What for to remember anything, if you have a computer? One can even forget that there exists a computer which is already growing into the system of conditioned (or unconditioned) reflexes of humanity and of a human being. Just as a person treating himself according to a medical reference book can die because of a misprint, so can a mathematically illiterate user of a computer provoke a catastrophe because of a decimal point placed improperly in a decimal fraction.

"Left hemisphere mathematics" and computerization of education are a poisonous mixture contributing to the destruction of psychological (not only psychological) health of schoolchildren.

Under the influence of new and clearly aggressive information environment, man is changing his biological nature and in place of "homo sapiens", "homo computeric" is coming into existence. (Indications of this new species can be seen in the examples given at the beginning.) I am afraid, however, that adaptability and evolutional possibilities of human race are inconsistent with the force of influence of the environment and the rate of its change achieved by the end of the 20th century.

"Internet" has become the basic global idea nowadays. The "world web" is predicted to embrace all educational space and force out both—the teacher and the school, the professor and university. (By the way, if there is the "web" there should be "flies" and "spiders".)

I remember an occurrence. Once at leisure time my wife and I happened to watch an abnormal rooster. "One cannot expect too much of him since he was grown in the incubator", concluded the rooster's owner philosophically. One can predict or foresee what might happen to the human race if it is brought up and educated on the Internet, but it is really terrifying to think about it.

Satisfaction with a minimum of physical necessities is enough to have a feeling of complete happiness during oligophrenia. One of the symptoms of intellectual degradation is a feeling of absolute self-satisfaction. It is difficult to cure a sick man if illness gives him pleasure. And is it necessary?

The conclusion. Summarizing the preceding, I would like to say that mathematicians are at fault before society to some extent. But their fault is infinitesimal in comparison with considerable advances of mankind which have been made owing to mathematics. It does not correspond either to persecutions and accusations to which mathematics is subjected by controlled mass media and supposedly public opinion.

And still, as it is customary in our country, we are optimists. And in this op-

timistic spirit I would like to conclude my article and answer briefly the question: Why do we need mathematics and mathematical education in the 21th century? Undeniably, mathematical education solves many educational problems of exceptional importance. But I will single out a problem which is now becoming nearly the most important one.

The main goal of mathematical education in the 21th century will be upbringing of intellectually and psychologically (even physiologically) healthy generation. There might be an objection to it: What mental health are you talking about, when any professional mathematician is the object of the longed-for dream of a psychiatrist? It is appropriate at this point to make the following analogy. Mathematical education and mathematical science are related as physical culture and sport. Physical training is useful and necessary for the normal development and functioning of an ordinary man. Professional sport is often the cause of a loss of health. But professional sport reveals boundless possibilities of a human body. Professional mathematics shows unbounded potentialities of human intellect.

Qualitative nutrition is required to maintain normal development of a child. Various intellectual food is essential for normal intellectual advancement. Today, mathematics, especially geometry, is one of the "ecologically safe" and valuable product used in the educational system. But mathematics is a product which must be cooked by a very skilful culinary specialist. Otherwise, it may not only lose its nutritious qualities, but do harm to the whole organism as well.

I should finish at this point, but nevertheless I will add in particular a few words about the role of geometry. I am deeply convinced that geometry possesses a lot of psycho- , physio- and simply therapeutic possibilities which have not been studied properly, or, more precisely, they have not been investigated at all. The artists Komar and Melamid have a collection of etchings which display different geometric figures which these artists recommend as medicine. ("Do you have a headache? Don't take aspirin. Our circle will help you.") Despite conjuncture which is typical for this famous tandem of artists it is missing from this particular collection. The theme came out from their subconsciousness and intuition and we must do justice to these artists it is well developed.

Therefore, study geometry and be well.

P.S. I would like to recall one story, whether a parable or an anecdote, whether anti-Semitic or on the contrary.

A base wave is approaching a little village located in the valley which will cover all the villagers with water in two hours. What is the response of holy pastors depending on different religions? A Christian priest would pray and absolve believers sins. A mullah would perform rituals in according with the laws of Islam. As for a rabbi, he would address the villagers: "Jews, you have two hours left to learn how to survive under water."

Maybe there is another way to survive!

Igor F. Sharygin, Moscow Centre for Continuous Mathematical Education